Let's Multiply—Maya Style!

Summary of Activity

This lesson allows students to perform Maya-style multiplication as a follow-up activity to the interactive *Maya Math* interactive tool available on the *Living Maya Time* website. Students will be able to do Maya-style math on a Base-10 grid using beans, sticks and shells.

Living Maya Time Website Connections

Students should engage with the following resources from the *Living Maya Time* website prior to this activity:

- The entire *Maya Sun* section and, in particular, use the *Maya Math* interactive tool.

Objectives

Students will:

- Learn how to visually identify the Maya number system.
- Understand Base-10 and Base-20 place-value.
- Be able to represent numbers in Base-10 up to 100,000 using beans, sticks, and shells on a square grid.
- Be able to represent numbers in Base-20 up to 160,000 using beans, sticks, and shells on a square grid.
- Be able to do addition and subtraction using the interactive tool in the Maya Math section of the website.
- Be able to do multiplication in Base-10 using beans, sticks, and shells on a square grid.
- Optional Extension: Be able to do multiplication in Base-20 using beans, sticks, and shells on a square grid.

Grade Level

Grades 7-8

National Council of Teachers of Mathematics Standards Addressed

- Number Systems
- Place Value

Duration of Activity

- 1 class period

Student Prerequisites

Students should:

- Have a basic understanding of place value.
- Explore the *Living Maya Time* website in advance of doing the activity.

Materials

- Butcher paper where students can draw square grids in Base-10 and Base-20
- Markers
- Toothpicks (3 boxes), beans (5 bags), and shells (20)

Teacher Preparation

Be familiar with the Maya number system (beans, sticks, and shells) from 0 to 19. Be familiar with place value in the decimal system. Be able to write numbers up to 100,000 in the decimal system using beans, sticks, and shells on a square grid in a row, then column, fashion, and similarly up to 160,000 in the vigesimal system. Become familiar with basic addition, subtraction, and multiplication on a square grid by using the interactive *Maya Math* tool on the website and by studying the examples given in this lesson.

Procedure

- Divide students into groups of 4.
- Support the hands-on activity with the interactive tool on the website, projecting the screen for all to see if possible.
- Refer to the examples at the bottom of this lesson to help students understand the basic concepts.
- Begin the discussion of number systems by having students write numbers from 0 to 9 in any number system they are familiar with (Arabic is our standard, but some students may be familiar with Roman Numerals, etc.).
- Present the Maya numbering system from 0 to 19 using toothpicks, beans, and shells (shell for "0 or completion," toothpick for "5" and bean for "1"). Distribute materials to students.
- Have the students draw several Base-10 and Base-20 grids on the butcher paper.
- Have students call out random numbers from zero to 20 and have students represent these numbers on their grids using the beans, sticks, and shells.
- Have students begin with Base-10, and write numbers in columns up to 100,000.
- Demonstrate simple addition in base-10, using the interactive tool and the paper grid.
- Have students compare and contrast our standard way of doing arithmetic with this grid-based way.
- Repeat addition and subtraction exercises, but now using the base-20 grid.
- Finish by performing Base-10 multiplication on a square grid using beans, sticks, and shells.
- As an optional extension, have students perform the same multiplication exercises using Base-20.

Assessment

The group work on the grid can serve as an assessment of student work. Ask students to discuss and share how they think the ancient Maya could have used this system to perform division.

ARABIC NUMBERS									
0 or completion	1	2	3	4	5	6	7	8	9

PLACE VALUE IN BASE-10						
Arabic number place values increase by a factor of 10 from right to left.						
← ... and so on	100,000s	10,000s	1,000s	100s	10s	1s

MAYA NUMBERS				
0 or completion	1	2	3	4
5	6	7	8	9
10	11	12	13	14
15	16	17	18	19

PLACE VALUE IN BASE-20

Maya number place values increase by a factor of 20 from bottom to top.

	... and so on
	160,000s
	8,000s
	400s
	20s
	1s

EXAMPLE: Writing the number 20 in Base-10

1,000s	100s	10s	1s
		2	0

$$2 \times 10 = 20 \qquad 0 \times 1 = 0 \qquad \begin{array}{r} 20 \\ + 0 \\ \hline = \\ 20 \end{array}$$

EXAMPLE: Writing the number 20 in Base-20

400s
20s
1s

1 x 20 = 20

0 x 1 = 0

_____

20 + 0 = 20

MULTIPLICATION GRID IN BASE-10							
		F	E	D	C	B	A
				1000s	100s	10s	1s
6							
5							
4	1000s						
3	100s						
2	10s						
1	1s						

Step 1: Have each group of 4 students draw a grid as in Figure 1 on the butcher paper

		F	E	D	C	B	A
EXAMPLE 1 **MULTIPLYING 12 X 12 IN BASE-10 USING MAYA NUMBERS** **Setting Up the Problem**							
				1000s	100s	10s	1s
6						●	●●
5							
4	1000s						
3	100s						
2	10s	●					
1	1s	●●					

Step 2: Give the students a simple example, such as multiplying 12 x 12 = 144.

Step 3: Pass around handfuls of beans, sticks, and a few shells to each group.

Step 4: Have the students set up the problem as shown above, using beans to place one unit in the 10s place box and two units in the 1s place box. Note: Students can start by placing 2 sticks and two beans in the 1s place, then reduce the answer by taking the "10" in the 1s place box and replacing the 2 sticks by 1 bean in the 10s place box.

		F	E	D	C	B	A
EXAMPLE 1 MULTIPLYING 12 X 12 IN BASE-10 USING MAYA NUMBERS Cross Multiplication							
				1000s	100s	10s	1s
6						●	●●
5							
4	1000s						
3	100s						
2	10s	●				●	●●
1	1s	●●				●●	●●●●

Step 5: Instruct the students to do all the cross multiplications:

- F1 X A6 and place the answer in A1
- F2 X A6 and place the answer in A2
- F1 x B6 and place the answer in B1
- F2 X B6 and place the answer in B2

		F	E	D	C	B	A
				1000s	100s	10s	1s
6						•	••
5							
4	1000s						
3	100s						
2	10s	•				•	••
1	1s	••				••	••••

EXAMPLE 1
MULTIPLYING 12 X 12 IN BASE-10 USING MAYA NUMBERS
Shift and Add

Step 6: Instruct the students to "shift and add:"

- Move all items from the left column to the right column, at a diagonal (shift and add)

		F	E	D	C	B	A
EXAMPLE 1 **MULTIPLYING 12 X 12 IN BASE-10 USING MAYA NUMBERS** **Final Answer**							
				1000s	100s	10s	1s
6						●	●●
5							
4	1000s						
3	100s						●
2	10s	●					●●●●
1	1s	●●					●●●●

Step 7: Reduce the answer, if needed, using the Reduction Lesson for Addition in the *Maya Math* section of the *Living Maya Time* website.

- The answer is 144, because there is a bean in the 100s place box, 4 beans in the 10s place box, and four beans in the 1s place box, which make 100 + 40 + 4 = 144.

EXAMPLE 2
MULTIPLYING 35 X 29 IN BASE-10 USING MAYA NUMBERS
Setting Up the Problem

		F	E	D	C	B	A
				1000s	100s	10s	1s
6						●●	●●● ❙
5							
4	1000s						
3	100s						
2	10s	●●●					
1	1s	▬					

This example illustrates a more complex case that requires more extensive use of the grid.

Step 1: Have the students set up the problem as shown above. To write "35," have them place three beans in the 10s place box and 1 stick in the 1s place box. Note: Students can start by placing five beans in the 1s place box and then reduce the answer by replacing the 5 beans by 1 stick. To write "29," have them place two beans in the 10s place box and 1 stick and four beans in the 1s place box. Note: Students can start by placing nine beans in the 1s place box and then reduce the answer by replacing 5 of the beans by 1 stick.

EXAMPLE 2

EXAMPLE 2
MULTIPLYING 35 X 29 IN BASE-10 USING MAYA NUMBERS
Cross Multiplication

		F	E	D	C	B	A
				1000s	100s	10s	1s
6						••• (3 dots)	••• (4 dots) ❘
5							
4	1000s						
3	100s						•• (2 green dots)
2	10s	••• (3 dots)				•• ▬ (2 dots over bar)	•• (2 green dots) / ▬ green bar / •••• (4 red dots)
1	1s	▬ (bar)				⬭ (zero/shell)	▬ (red bar)

Step 2: Instruct the students to do all the cross multiplications:

- Multiply F1 x B6 and place the answer in B1. Note that the result, 10, is more than 9 thus we need to write it as by placing 1 bean in the 10s place box and a zero in the 1s place box.
- Multiply F2 X B6 and place the answer in B2
- Multiply F1 X A6 and place the answer in A1 (red); Note that the answer, 45, does not fit in the A1 box because it is more than 9. Thus you need to place the 5 in the 1s box and the 40 in the 10s box (4 beans).
- Multiply F2 X A6 and place the answer in A2 (green); Note that the answer, 27, does not fit in the A2 box, because it is more than 9. Because the resultant cross product needs to be placed in the A2 position, this box now becomes the 1s box for the green product, and A3 box becomes the 10s box for the green product. Thus you need to place the 7 in the 1s box (now A2) and the 20 (two beans) in the 10s box (now A3).

		F	E	D	C	B	A
EXAMPLE 2 **MULTIPLYING 35 X 29 IN BASE-10 USING MAYA NUMBERS** **Shift and Add**							
				1000s	100s	10s	1s
6						••	(••• + bar)
5							
4	1000s						
3	100s						•• (green)
2	10s	•••				•• / —	green bar / •••• (red)
1	1s	—				⬭	— (red)

Step 3: Instruct the students to "shift and add:"

- Move all items from the left column to the right column at a diagonal (shift and add).

EXAMPLE 2

EXAMPLE 2
MULTIPLYING 35 X 29 IN BASE-10 USING MAYA NUMBERS
Shift and Add – Preliminary Answer

		F	E	D (1000s)	C (100s)	B (10s)	A (1s)
6						●●	●●●● │
5							
4	1000s						
3	100s						●●●●● ▬
2	10s	●●●					●●● ● ⬭
1	1s	▬					▬

Step 4: The answer above is preliminary because there are more than 9 units in the 10s box. Use the Reduction Lesson for Addition in the *Maya Math* section of the *Living Maya Time* website to reduce the answer, making sure that there are fewer than 10 units in each box.

EXAMPLE 2
MULTIPLYING 35 X 29 IN BASE-10 USING MAYA NUMBERS
Final Answer

		F	E	D	C	B	A
				1000s	100s	10s	1s
6						●●	●●● |
5							
4	1000s						●
3	100s						⬭
2	10s	●●●					●
1	1s	▬					▬

Step 5: The answer is 1015, because there is a bean in the 1000s place box, a complete box (with a zero) in the 100s place box, a bean in the 10s place box, and a 5 in the 1s box, which make 1000 + 0 + 10 +5 = 1015.